

Teichmüller Theory and Beyond

February 17 – 21, 2025

Building 28, Room 102
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Speakers

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Sebastian Heller	<i>Beijing Institute of Mathematical Sciences and Applications</i>
Yi Huang	<i>Tsinghua University</i>
Tsukasa Ishibashi	<i>Tohoku University</i>
Hongtaek Jung	<i>Seoul National University</i>
Sungwoon Kim	<i>Jeju National University</i>
Youngju Kim	<i>Konkuk University</i>
Didac Martinez-Granado	<i>University of Luxembourg/National University of Singapore</i>
Hidetoshi Masai	<i>Musashino Art University</i>
Yosuke Morita	<i>Kyushu University</i>
Huiping Pan	<i>South China University of Technology</i>
Weixu Su	<i>Sun Yat-sen University</i>
Zhe Sun	<i>University of Science and Technology of China</i>
Andrea Tamburelli	<i>University of Pisa</i>
Binbin Xu	<i>Nankai University</i>
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Monday, 17 February 2025

- 10:20 – 10:50 Coffee
- 10:50 – 11:00 Opening
- 11:00 – 12:00 Exotic proper actions on reductive homogeneous spaces via convex cocompact representations
Yosuke Morita, Kyushu University
- I will give an example of a reductive homogeneous space G/H which admits proper actions of discrete subgroups of G isomorphic to cocompact lattices of $O(n, 1)$ ($n = 2, 3, 4$) but does not admit proper actions of non-compact semisimple Lie subgroups of G . The existence of such G/H was previously not known even for $n = 2$. The construction of proper actions is based on Guéritaud-Kassel's work on convex cocompact subgroups of $O(n, 1)$ and Danciger-Guéritaud-Kassel's work on right-angled Coxeter groups. This is joint work with Maciej Bocheński.
- 12:00 – 14:00 Lunch
- 14:00 – 15:00 Length orders of curves on hyperbolic surfaces
Binbin Xu, Nankai University
- Let S be an oriented topological surface of finite type. Given a hyperbolic metric on S , there is an order among all homotopy classes of curves on S induced by comparing the lengths of their geodesic representatives. We call it the length order of curves induced by the given hyperbolic metric. In a collaboration with Hugo Parlier and Hanh Vo, we show that given any pair of distinct points in the Teichmüller space $\mathcal{T}(S)$ of S , there exist two homotopy classes of curves on S , such that the two given points of $\mathcal{T}(S)$ induce different length order on them. Hence the length orders of curves on S can determine points in $\mathcal{T}(S)$. This result is a generalization of a result of Greg McShane and Hugo Parlier. We also study the homotopy classes of curves whose length order never changes as the hyperbolic metric varies, and introduce a way to construct such examples.
- 15:00 – 15:30 Coffee
- 15:30 – 16:30 Generic properties of Hitchin representations
Hongtaek Jung, Seoul National University
- Let G be a split real form not of types A_n , D_{2n+1} , and E_6 and let J be the Jordan projection. Let H be the fundamental group of a closed orientable orbifold. We study the image of the Jordan projection $J(f) := \{J(f(x)) \mid f : H \rightarrow G \text{ Hitchin}, x \in H\}$. Given any hyperplane V in the maximal abelian subalgebra of G , we show that, under mild conditions on orbifolds, $J(f)$ is disjoint from V for generic G -Hitchin representations. As an application, we prove that generic orbifold G -Hitchin representations are strongly dense in the sense of Breuillard–Green–Guralnick–Tao. This shows that there are plenty of strongly dense orbifold subgroups in G , partially generalizing a result of Breuillard–Guralnick–Larsen.
- 16:30 – 17:00 Coffee
- 17:00 – 18:00 Para-complex geometry of cyclic $SL(2m+1, \mathbb{R})$ -Higgs bundles
Andrea Tamburelli, University of Pisa
- Para-complex hyperbolic space is the para-complex analogue of the more familiar complex hyperbolic space and resembles in many ways the classical hyperbolic space. In this talk I will explain how we can use para-complex geometry to construct a 1:1 correspondence between cyclic $SL(2m+1, \mathbb{R})$ -Higgs bundles and a special class of minimal surfaces in the $2m$ -dimensional para-complex hyperbolic space. This is joint work with Nicholas Runge.

Tuesday, 18 February 2025

- 09:30 – 10:30 Existence of holomorphic curves into $SL(2, \mathbb{C})$ modulo a cocompact lattice
Sebastian Heller, Beijing Institute of Mathematical Sciences and Applications
We prove the existence of a pair (Σ, Γ) , where Σ is a compact Riemann surface with $\text{genus}(\Sigma) \geq 2$, and $\Gamma \subset SL(2, \mathbb{C})$ is a cocompact lattice, such that there is a generically injective holomorphic map $\Sigma \rightarrow SL(2, \mathbb{C})/\Gamma$. This gives an affirmative answer to a question raised by Huckleberry–Winkelmann and by Ghys.
- 10:30 – 11:00 Coffee
- 11:00 – 12:00 Patterson-Sullivan measures for relative Anosov groups
Tengren Zhang, National University of Singapore
Relative Anosov groups are a class of subgroups of a semisimple Lie group G that include all Anosov subgroups, and all geometrically finite subgroups when G has rank one. We prove that under some mild conditions on G , the Poincaré series associated to a relative Anosov subgroup (and a linear function on the Cartan subspace of G) diverges at its critical exponent if its critical exponent is finite. As a consequence of this, we deduce uniqueness and ergodicity results for the associated Patterson-Sullivan measure whose dimension is the critical exponent. This is joint work with Richard Canary and Andrew Zimmer.
- 12:00 – 14:00 Lunch
- 14:00 – 15:00 Tubes in complex hyperbolic manifolds
Youngju Kim, Konkuk University
We will talk about a tubular neighborhood theorem for an embedded complex geodesic in a complex hyperbolic 2-manifold where the width of the tube depends only on the Euler characteristic of the embedded complex geodesic. We give an explicit estimate for this width. We supply two applications of the tubular neighborhood theorem, the first is a lower volume bound for such manifolds. The second is an upper bound on the first eigenvalue of the Laplacian in terms of the geometry of the manifold.
- 15:00 – 15:30 Coffee
- 15:30 – 16:30 A sibling of the Teichmüller distance
Hidetoshi Masai, Musashino Art University
We first discuss a general procedure to generate distances and horofunctions via a function of type $F : X \times Y \rightarrow \mathbb{R}$. Then, in this talk, we focus on the Extremal length functional extended on the space of geodesic currents by Martínez-Granado and D. Thurston. We then construct a distance on the Teichmüller space which is a sibling of the Teichmüller distance.
- 16:30 – 17:00 Coffee
- 17:00 – 18:00 Envelopes of the Thurston metric on Teichmüller space
Huiping Pan, South China University of Technology
The Thurston (asymmetric) metric on Teichmüller space is geodesic but not uniquely geodesic. The union of geodesics from the initial point to the terminal point is called the envelope. In this talk, we will discuss the structure of envelopes. This is based on a joint work with Michael Wolf (arXiv:2401.06607).
- 19:00 – Social Dinner

Wednesday, 19 February 2025

09:30 – 10:30 Exponential volumes of moduli spaces of hyperbolic surfaces

Zhe Sun, University of Science and Technology of China

Mirzakhani found a remarkable recursive formula for the volumes of the moduli spaces of the hyperbolic surfaces with geodesic boundary, and the recursive formula plays very important role in several areas of mathematics: topological recursion, random hyperbolic surfaces etc. We consider some more general moduli spaces $\mathcal{M}_S(K, L)$ where the hyperbolic surfaces would have crown ends and horocycle decorations at each ideal points. But the volume of the space $\mathcal{M}_S(K, L)$ is infinite when S has the crown ends. To fix this problem, we introduce the exponential volume form given by the volume form multiplied by the exponent of a canonical function on $\mathcal{M}_S(K, L)$. We show that the exponential volume is finite. And we prove the recursion formulas for the exponential volumes, generalising Mirzakhani's recursions for the volumes of moduli spaces of hyperbolic surfaces. We expect the exponential volumes are relevant to the open string theory. This is a joint work with Alexander Goncharov.

10:30 – 11:00 Coffee

11:00 – 12:00 Bending Teichmüller spaces and character varieties

Shinpei Baba, Osaka University

A hyperbolic structure on a (closed) oriented surface corresponds to a discrete faithful representation of the surface group into $\mathrm{PSL}(2, \mathbb{R})$. We can “bend” this real representation along a measured geodesic lamination, and obtain a representation of the surface group into $\mathrm{PSL}(2, \mathbb{C})$. Moreover, by varying the hyperbolic structure while fixing the bending measured lamination, we obtain a real analytic mapping from the Teichmüller space into the $\mathrm{PSL}(2, \mathbb{C})$ -character variety of the surface. In this talk, we explain some properties of this real analytic mapping which resemble the holonomy variety of $\mathbb{CP}^1$ -structures. We also explain a complexification of the real analytic mapping in the space of representations of the surface group into $\mathrm{PSL}(2, \mathbb{C}) \times \mathrm{PSL}(2, \mathbb{C})$.

12:00 – 14:00 Lunch

14:00 – Free afternoon

Thursday, 20 February 2025

- 09:30 – 10:30 Skein and cluster quantizations of the moduli space of decorated G -local systems
Tsukasa Ishibashi, Tohoku University
- For a compact surface with marked points, Fock and Goncharov introduced the moduli space of G -local systems on the surface, enriched by decoration data at the marked points. This moduli space features a special atlas called a cluster K_2 structure, whose positive real part is the decorated version of higher Teichmüller space. When all the marked points lie on the boundary, the cluster K_2 structure gives rise to a deformation quantization via the combinatorial framework of quantum cluster algebra (Berenstein—Zelevinsky, Goncharov—Shen). In this talk, I will explain the connection between this framework and the skein algebra, which provides a diagrammatic approach to quantizing the G -character varieties.
- 10:30 – 11:00 Coffee
- 11:00 – 12:00 Semistability of the boundary actions of uniform lattices
Sungwoon Kim, Jeju National University
- Kapovich, Kim and Lee proved that the boundary actions of uniform lattices in semisimple Lie groups are stable in the diffeomorphism group and then, Connell, Islam, Nguyen and Spatzier showed that the boundary actions are semistable in the homeomorphism group. We give an alternative proof of the semistability of the boundary actions of uniform lattices in the homeomorphism group by extending the method of Kapovich, Kim and Lee to the homeomorphism group.
- 12:00 – 14:00 Lunch
- 14:00 – 15:00 Coulomb branches and character varieties of surfaces
Dylan G.L. Allegretti, Tsinghua University
- I will describe a new way of studying the $\mathrm{SL}(2, \mathbb{C})$ -character variety of a surface using the notion of a K -theoretic Coulomb branch introduced by Braverman, Finkelberg, and Nakajima in geometric representation theory. This talk is based on joint work with Peng Shan.
- 15:00 – 15:30 Coffee
- 15:30 – 16:30 Volume function of unit balls associated to quadratic differentials
Weixu Su, Sun Yat-sen University
- Given a holomorphic quadratic differential on a closed Riemann surface, there is a unit ball of the measured lamination space determined by the associated flat metric. The Thurston volume of the unit ball define a function on the moduli space. We show that such a volume function is improper and characterize when it tends to infinity. We also give upper bound of the volume function and show that its square root is integrable on the moduli space (with respect to the Masur-Veech measure). This is a joint work with Shenxing Zhang (Fudan University).
- 16:30 – 17:00 Coffee
- 17:00 – 18:00 The Earthquake Metric
Yi Huang, Tsinghua University
- We introduce the earthquake metric on Teichmüller space, and some of its properties. As well as its connections to Thurston's (Lipschitz) metric. This is joint work with Ken'ichi Ohshika, Huiping Pan and Athanase Papadopoulos.

Friday, 21 February 2025

09:30 – 10:30 Metrics and reparameterizations for hyperbolic groups

Didac Martinez-Granado, University of Luxembourg/National University of Singapore

I will present the “space of metrics of a group”, a metric space parameterizing the geometric actions of an arbitrary hyperbolic group on Gromov hyperbolic spaces. Even for the surface group case, this space is much larger than the classical Teichmüller space, encompassing negatively curved Riemannian metrics, geodesic currents, Anosov representations, random walks, and more. I will explore how Green metrics—those associated with admissible random walks on the group—are dense in the space of metrics. As an application, I will show that, for a closed negatively curved Riemannian manifold, there exists a natural isometry between the closure of the space of reparameterizations of the geodesic flow and the closure of the space of metrics. This is joint work with Stephen Cantrell and Eduardo Reyes.

10:30 – 11:00 Coffee

11:00 – 12:00 Affine laminations and coaffine representations

James Farre, Max Planck Institute for Mathematics in the Sciences

A surface subgroup Γ of $\mathrm{PSL}(4, \mathbb{R})$ is convex cocompact if it preserves a nice convex set in projective 3-space on which it acts with compact quotient. The cases that Γ preserves a non-degenerate quadratic form on $\mathbb{R}^4$ have been well-studied and correspond to “quasi-Fuchsian” 3-manifolds locally modeled on hyperbolic or anti de Sitter 3-space. In this talk, we study surface group actions on coaffine 3-space—another subgeometry of projective geometry. After an introduction to coaffine geometry, I will describe the structure of the boundary of the convex core associated to a convex cocompact surface group action on coaffine space: it is a 2-dimensional convex projective structure bent along a geodesic lamination according to an affine measure. This is joint work with Martin Bobb.

12:00 – 14:00 Lunch